QUIZ #6 – Solutions
Each problem is worth 5 points

15 points total

1.

$$\frac{x^2}{3-4x} = -\frac{x}{4} - \frac{3}{16} + \frac{9/16}{3-4x} = -\frac{1}{4}(x-2) - \frac{11}{16} + \frac{9/16}{-5-4(x-2)} = -\frac{11}{16} - \frac{1}{4}(x-2) - \frac{9/80}{1+\frac{4(x-2)}{5}}$$

$$= -\frac{11}{16} - \frac{1}{4}(x-2) - \frac{9}{80}\sum_{n=0}^{\infty}\left[-\frac{4}{5}(x-2)\right]^n$$

$$= -\frac{4}{5} - \frac{4}{25}(x-2) + \sum_{n=2}^{\infty}\frac{9(-1)^{n+1}4^{n-2}}{5^{n+1}}(x-2)^n, \quad \left|-\frac{4(x-2)}{5}\right| < 1 \implies \frac{3}{4} < x < \frac{13}{4}$$

2.

The radius of convergence of the series is $R = 1$. If we set $S(x) = \sum_{n=0}^{\infty}\frac{1}{n+1}x^n$, then $x\,S(x) = \sum_{n=0}^{\infty}\frac{1}{n+1}x^{n+1}$. Term-by-term differentiation gives $\frac{d}{dx}[x\,S(x)] = \sum_{n=0}^{\infty}x^n = \frac{1}{1-x}$, since the series is geometric. We now antidifferentiate,

$$x\,S(x) = \int \frac{1}{1-x}dx = -\ln(1-x) + C.$$

Since $S(0) = 1$, it follows that $C = 0$, and $S(x) = -\frac{1}{x}\ln(1-x)$.

3.

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \to 0} 1/x\{(1 + 1/2\,x - 1/8\,x^2 + \dots) - 1\}$$

$$= \lim_{x \to 0} \{1/2 - 1/8\,x + 1/16\,x^2 - \dots\} = 1/2$$